The surface Plasmon's frequencies of two Metallic Nanospheres by Bloch-Jensen Hydrodynamical Model

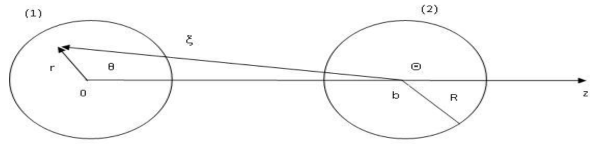
M.Mirasmouri[1],F.Ebrahimi[2],V.Ebrahimi[3]
m.mirasmouri@uoz.ac.ir
[1] Department of Physics, Faculty of Sciences, university of zabol,zabol,Iran
[2] Department of Physics, Faculty of Sciences, Shahid Beheshti University, Evin, Tehran, Iran
[3] Department of Physics, Faculty of Sciences, university of Tabriz,Tabriz,Iran

The wave guides and optical fibers have long been known to transmit light and electromagnetic fields in large dimensions. Recently, surface plasmons, that are collective plasma oscillations of valence electrons at metal surfaces,have been introduced as an entity that are able to guide light on the surfaces of the metal and to concentrate light in subwavelength volumes.It has been found that periodic array of metallic nanospheres, could be able to enhanced the light transmission, and guiding light at nanoscale[1-2].The coupling between two nanoparticle in these devices is very important.

The Bloch-Gensen hydrodynamical method has been used for computing surface plasmons frequencies of a single metallic nanosphere[3]. In this research,we compute the surface plasmons frequencies of two nanospheres by bloch-gensen hydrodynamical model *for the first time*. The results show that the plasmons frequencies are depend explicitly on the nanoparticle radius. The hydrodynamical model contains the entire pole spectrum automatically, so it is more exactly than the other computational methods.

The surface Plasmon's frequencies of two nanospheres

We consider two nanospheres that they closely spaced in vacuum and the space between them is b in the z direction. According to figure1The center of sphere 1 is in z=0 and sphere 2 is in z=b.



The electron density for sphere 1 and sphere 2 is[3],

$$n_1(r,\theta,\varphi,t)=\sum_{lm}A_{lm}^{(1)}i_l(kr)y_{lm}(\theta,\varphi)e^{-i\omega_l t}+H.C,r\le R \quad n_2(\xi,\Theta,\varphi,t)=\sum_{lm}A_{lm}^{(2)}i_l(k\xi)y_{lm}(\Theta,\varphi)e^{-i\omega_l t}+H.C,\xi\le R \quad (1)$$

Where (r,θ,φ) and (ξ,Θ,ϕ) are polar coordinates of sphere 1 and sphere2 respectively, $i_l(kr)$ is modified Bessel function and $y_{l,m}$ is spherical harmonics. In order to surface plasmon's frequencies computation the electric potentials are calculated and then the boundary condition is applied, that is the radial electric current density at the surface is zero $(j_r|_{r=R}=0)$ It should be noted that the total potential inside of sphere1 is due to the electron densities of spheres1,2, so the electrical potential resulting of electron density of sphere1 inside of it is [9],

$$\phi_1^1(r,\theta,\varphi,t)=\sum_{lm}A_{lm}^{(1)}\frac{4\pi}{k^2}(i_l(kr)-\frac{kR}{2l+1}i_{l-1}(kR)(\frac{r}{R})^l)y_{lm}(\theta,\varphi)e^{-i\omega_l t}+H.C \quad (2)$$

and the electron density of sphere2 produce electrical potential inside of sphere1 that is [3],

$$\phi_1^2(\xi,\Theta,\varphi,t)=-4\pi\sum_{lm}\frac{A_{lm}^{(2)}}{2l+1}\frac{y_{lm}(\Theta,\varphi)}{\xi^{l+1}}\frac{R^{l+2}}{k}i_{l+1}(kR)e^{-i\omega_l t}+H.C \quad (3)$$

This two potential should be written in the same coordinate so we use $\vec{\xi}=\vec{r}-\vec{b}$ [4] and so the total potential inside of sphere 1 is,

$$\begin{aligned}\phi_1^T &= \phi_1^1+\phi_1^2= \\ &\sum_{lm}A_{lm}^{(1)}\frac{4\pi}{k^2}(i_l(kr)-\frac{kR}{2l+1}i_{l-1}(kR)(\frac{r}{R})^l)y_{lm}(\theta,\varphi) \\ &-\sum_{lm}A_{lm}^{(2)}\frac{4\pi}{k^2}(-1)^{l+m}H(l',l,m)\frac{R^{l+1}}{b^{l'+1}}y_{l'-l,m}(\theta,\varphi)(kR)r^{l'-l}i_{l+1}(kR)\end{aligned} \quad (4)$$

Where, $H(l',l,m)=\left(\frac{2l+1}{(2(l'-l)+1)}\frac{l'!\,l'!}{(l+m)!(l'-l+m)!(l-m)!(l'-l-m)!}\right)^{1/2}$ (5)

Now the boundary condition is applied and by using the equation (1),

$$j_r\big|_{r=R}=0\rightarrow\frac{\partial\phi_1^T(r,\theta,\varphi)}{\partial r}\Big|_{r=R}=\frac{\beta^2}{n_0}\frac{\partial n_1(r,\theta,\varphi)}{\partial r}\Big|_{r=R} \quad (6)$$

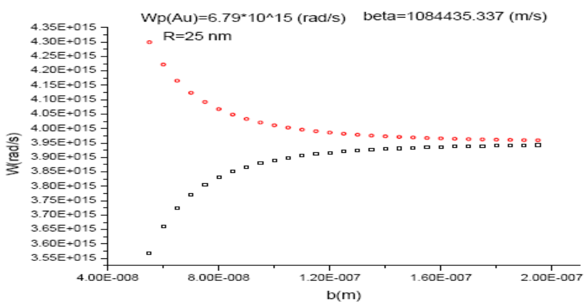
by inserting (1) and (4) in the equation (6) and some algebra the following relation is achieved

$$A_{l,m}^{(1)}\left[\left(\frac{\omega_p^2}{k^2}-\beta^2\right)\frac{k(l+1)}{2l+1}i_{l+1}(kR)-\frac{\beta^2 kl}{2l+1}i_{l-1}(kR)\right]$$
$$=\sum_{\substack{l'\\ l'\geq l+|m|}}A_{l'-l,m}^{(2)}\frac{\omega_p^2}{k}(-1)^{l'-l+m}H(l',l,m)\left(\frac{R}{b}\right)^{l'+1}l\,i_{l'-l+1}(kR) \quad (7)$$

In order to applying the boundary condition $j_\xi\big|_{\xi=R}=0$ we need to write the potential of sphere1 with respect of sphere2 coordinates, The computations are like the sphere1 computations and by doing them the relation (8) is obtained for sphere2,

$$A_{l'',m''}^{(2)}\left[\left(\frac{\omega_p^2}{k^2}-\beta^2\right)\frac{k(l+1)}{2l+1}i_{l+1}(kR)-\frac{\beta^2 kl}{2l+1}i_{l-1}(kR)\right]=\sum_{\substack{l'\\ l'\geq l+|m|}}A_{l'-l,m}^{(1)}\frac{\omega_p^2}{k}(-1)^{2l'-l+m}H(l',l,m)\left(\frac{R}{b}\right)^{l'+1}l\,i_{l'-l+1}(kR) \quad (8)$$

It is necessary to computed matrix density coefficients and find the roots of its determinant, that they are surface plasmons modes of these two metallic spheres. By assuming that the radiuses of nanospheres are 25 nm. And by computing the roots of matrix coefficients, it is observed that two roots exist for a given b, these roots are surface Plasmon's modes for these two nanospheres, and the frequencies are plotted in below fig. The results are shown that if the space between this two sphere increased, the frequencies get close together and are equal to single sphere surface plasmon frequency in the state $l=1$. It is observed that surface plasmon frequency of a sphere with radius 20nm and $\beta=1084435.337\,m/s$ and $\omega_p=6.79\times10^{15}\,rad/s$ is equal to $\omega=3.95827\times10^{15}\,rad/s$.



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